King Faisal
PRIZE

King Faisal PRIZE

Mathematical Foundations with Lasting Impact: The Scientific Legacy of Carlos Kenig

Professor Najla Altwaijry
Department of Mathematics, College of Science, King Saud University

Abstract

Many modern scientific and technological advances rely on partial differential equations (PDEs) to describe change in space and time. Yet the equations that best model reality are often nonlinear, involve rough environments, and resist classical solution techniques. This article reviews, in accessible language, how the work of Carlos Kenig transformed the analysis of nonlinear PDEs and boundary phenomena, strengthened the mathematical foundations behind reliable modelling, and influenced applications ranging from wave propagation and fluid mechanics to inverse problems and emerging physics-informed machine learning. The discussion emphasises conceptual breakthroughs, long-term impact, and future scientific directions enabled by these ideas.

1 Introduction: Why Mathematics Matters Beyond Mathematics

Modern science and technology rely fundamentally on mathematics, often in ways that are invisible to the non-specialist. The accurate modelling of physical, biological, and technological systems — from fluid motion and wave propagation to signal transmission and medical imaging — depends on a precise mathematical description of how quantities evolve in space and time. At the heart of many such descriptions lie *partial differential equations* (PDEs), which serve as the primary language for expressing physical laws and dynamic processes [1, 2].

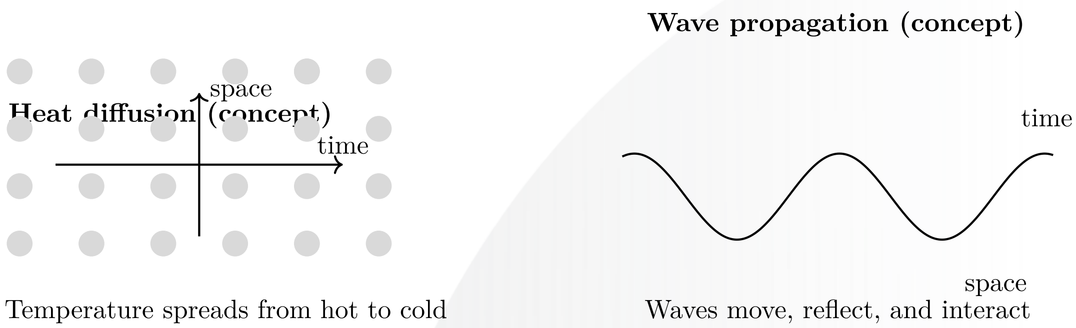
Despite their apparent abstraction, PDEs govern phenomena that shape everyday life. The equations describing heat flow underpin thermal engineering; wave equations govern acoustics, optics, and telecommunications; and fluid equations form the backbone of aerodynamics and climate modelling. The reliability of modern simulations, predictions, and designs depends critically on rigorous mathematical understanding: it clarifies what a model can (and cannot) predict, what data are needed, and which numerical outputs can be trusted.

The work of Carlos Kenig has played a central role in advancing this understanding. Over several decades, his research transformed the theory of nonlinear PDEs by introducing analytical frameworks capable of addressing complexity, irregularity, and instability — features inherent to real-world systems. Rather than focusing on isolated equations, Kenig developed methods that revealed deep structural properties common to broad classes of problems, reshaping how mathematicians approach some of the most difficult questions in analysis [3, 4].

What distinguishes Kenig's contributions is their enduring influence across disciplines. Ideas originating in his work are now embedded in the standard toolkit of modern analysis, enabling progress not only in pure mathematics but also in applied sciences where reliable modelling is indispensable. This combination of conceptual depth, methodological innovation, and long-term influence places his work among the most significant contributions to contemporary mathematical science.

2 Understanding Partial Differential Equations: Mathematics Behind Real-World Phenomena

Partial differential equations arise whenever one seeks to describe how a physical quantity changes with respect to both space and time. Unlike algebraic equations, which determine single numerical values, PDEs describe *fields*: quantities defined across a region, evolving over time. For example, a heat equation models how temperature varies throughout an object and changes over time, while a wave equation describes the propagation of sound or light through a medium [1].



(a) A PDE describes how a quantity (e.g., temperature) changes in both space and time

(b) Wave PDEs model motion of sound/light through complex environments

Figure 1: Illustrative diagrams for PDEs

Complexity, Nonlinearity, and Irregularity

Many PDEs encountered in applications are *nonlinear*, meaning the system's response is not proportional to the input. Nonlinearity is the mathematical signature of interaction: waves can reinforce or cancel; flows can become turbulent; and interfaces can move in ways that depend on the evolving solution itself. In such settings, small perturbations can produce large effects, and classical linear methods no longer apply [2].

A further challenge arises from *irregularity*. Natural systems rarely possess perfectly smooth boundaries or uniform material properties. Interfaces may deform, coefficients may vary abruptly, and measurements may be noisy. Consequently, solutions may fail to be smooth and can develop singularities — points at which classical descriptions break down. Determining whether solutions exist, whether they are unique, and whether they behave stably under perturbations becomes a fundamental scientific problem [5].

From Explicit Solutions to Structural Understanding

Historically, progress in PDE theory relied on obtaining explicit solutions for special equations under idealised assumptions. While successful in certain settings, this approach proved insufficient for realistic models. As applications grew more complex, mathematicians increasingly focused on *qualitative structure*: not an explicit formula, but guarantees about existence, uniqueness, regularity, and stability.

This shift led to questions such as: Under what minimal conditions do solutions exist? How regular are they near boundaries or interfaces? How sensitive are they to changes in data or geometry? Addressing these required new analytical tools capable of handling nonlinearity and roughness simultaneously.

It is within this intellectual landscape that the contributions of Carlos Kenig emerged as transformative. By combining ideas from harmonic analysis, geometric considerations, and PDE theory, Kenig helped establish unified frameworks for understanding broad classes of equations, laying groundwork for reliable mathematical modelling in both theory and practice [3, 6].

3 Transformative Contributions of Carlos Kenig

The influence of Carlos Kenig's work extends far beyond individual results. His research reshaped the modern theory of PDEs by addressing core difficulties that long limited progress: free boundaries, low regularity, and the interplay between local and global behaviour.

Free Boundary Problems: When the Unknown Is the Boundary Itself

In many physical systems, the boundary separating different phases or materials evolves as part of the system. Examples include the surface of a fluid, the moving front of melting ice, and interfaces in phase transitions. Such *free boundary problems* are among the most challenging in analysis because the geometry of the domain is unknown *a priori* and must be determined simultaneously with the solution.

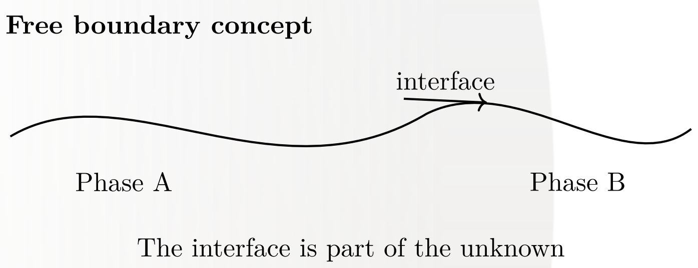


Figure 2: In free boundary problems, the evolving interface between phases must be determined together with the solution

Kenig's work, often in collaboration with leading mathematicians, introduced new methods to analyse the regularity and stability of free boundaries under minimal assumptions [7, 8]. These results demonstrated that even in irregular settings, free boundaries can exhibit strong structural properties. The insight that "order" can emerge from systems that appear unstable provided rigorous foundations for models used in fluid dynamics, materials science, and related fields.

Regularity Theory and Robustness

Another central theme of Kenig's research is *regularity theory*: understanding how smooth solutions are, even when data or environments are rough. Classical theory often required strong smoothness assumptions, limiting applicability. Kenig helped show that meaningful regularity can be established under far weaker conditions, greatly expanding the scope of PDE analysis [4, 6].

These advances have profound implications. They ensure that mathematical models remain reliable in realistic scenarios where perfect smoothness is unattainable, thereby strengthening the link between theory and application. In particular, regularity results often provide the backbone for numerical stability: they justify why discretisations converge and why computed quantities correspond to the intended physical behaviour.

Harmonic Analysis as a Unifying Framework

A defining feature of Kenig's work is the systematic use of *harmonic analysis* to study PDEs. By exploiting frequency-based techniques and multiscale analysis, he uncovered hidden structures within complex equations and resolved longstanding open problems [3, 9].

This synthesis unified previously disparate areas of mathematics and provided a common language for analysing waves, boundaries, and singularities. The resulting methods are now standard tools in modern analysis, influencing research across pure and applied mathematics.

4 From Theory to Real Life: Applications and Societal Impact

At first glance, Kenig's work may appear distant from everyday experience, rooted in abstract analysis. Yet the analytical frameworks he helped develop play a crucial role in ensuring that mathematical models of real-world phenomena are reliable, stable, and predictive. In many applied sciences, the difference between a useful model and a misleading one depends precisely on regularity, stability, and boundary behaviour.

Fluid Mechanics and Complex Flow

The equations governing fluid motion, such as the Euler and Navier–Stokes equations, describe phenomena ranging from airflow over aircraft wings to ocean currents and weather systems. These equations are highly nonlinear, and their solutions often involve irregular boundaries and interfaces. Kenig's contributions to boundary behaviour and low-regularity analysis support the mathematical foundations that underpin modern computational fluid dynamics [10].

Wave Propagation and Signal Transmission

Wave phenomena lie at the heart of acoustics, optics, and telecommunications. Kenig's work on elliptic boundary value problems and related analytical techniques informs how mathematicians and engineers analyse wave behaviour in complex environments [3, 11]. These ideas influence modelling of signal propagation in optical fibres, waveguides, and layered media, where small irregularities can significantly affect performance.

Medical Imaging and Inverse Problems

Medical imaging technologies such as ultrasound and tomography depend on *inverse problems*: determining internal structures from boundary measurements. These problems are delicate, as they can amplify noise and instability. Analytical tools developed within the broader PDE community, including those associated with Kenig's frameworks, support understanding of uniqueness and stability in inverse problems [12]. This theoretical foundation contributes to reliable reconstruction and hence to trustworthy diagnosis.

Materials Science and Phase Transitions

In materials science, PDEs model phase transitions, fracture propagation, and composite behaviour. Many processes involve evolving interfaces (free boundaries), precisely where Kenig's influence is prominent [7, 13]. By establishing conditions under which interfaces remain regular and predictable, these frameworks support the development of materials with tailored properties.

A Foundation for Emerging Technologies

As scientific modelling increasingly intersects with data-driven methods and artificial intelligence, PDE-based models remain indispensable for physical consistency and interpretability. Hybrid approaches that combine data with mathematical structure depend critically on robust analytical theory. In this context, Kenig's emphasis on structure and minimal assumptions continues to guide reliable modelling.

These applications depend indirectly but critically on the kind of foundational analysis developed by Kenig.

5 Long-Term Scientific Influence and Intellectual Legacy

The enduring impact of Carlos Kenig is measured not only by problems solved, but by how his ideas reshaped the intellectual landscape of modern analysis. Many central themes in contemporary PDEs — boundary regularity, multiscale analysis, stability under minimal assumptions, and the interaction between geometry and analysis — bear the imprint of his contributions [4, 3].

A defining aspect of his legacy is mentorship and intellectual leadership. Through supervision, collaboration, and sustained engagement with the global community, he helped transmit rigorous analytical thinking to successive generations. This propagation ensures that his contributions continue to evolve, rather than remaining confined to a fixed set of results.

6 Future Scientific Pathways Enabled by Kenig's Work

The long-term value of fundamental mathematics lies not only in immediate applications, but also in pathways it opens for future discovery. Kenig's analytical frameworks continue to shape emerging directions where complexity, uncertainty, and large-scale data intersect.

Data-Driven and Hybrid PDE Models

Modern modelling increasingly combines physical laws with data-driven techniques. In hybrid approaches, PDEs encode structure while data refine parameters or unresolved effects. The success of these models depends on rigorous foundations ensuring stability and interpretability — precisely the kind of robustness promoted by Kenig's approaches.

Physics-Informed Learning and Scientific Machine Learning

Physics-informed neural networks and related methods embed PDE constraints into learning algorithms, combining data with physical laws. Foundational understanding of well-posedness, regularity, and stability remains essential for knowing when such methods are reliable [14, 15].

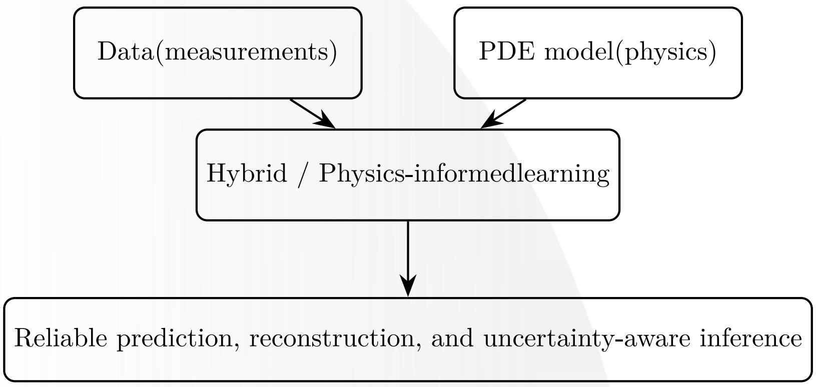


Figure 3: Conceptual view of physics-informed learning: combining data and PDE constraints to obtain reliable inference

Climate Science, Multiscale Systems, and Interfaces

Climate and geophysical models involve coupled PDE systems across scales. Analytical tools for boundary effects, interfaces, and stability remain central to improving interpretability and trustworthiness of these models [16].

Advanced Materials and Multiphase Systems

Future advances in energy-efficient materials and smart composites will rely on predictive models of evolving interfaces and heterogeneous structures. Frameworks for free boundaries and low regularity continue to guide the balance between complexity and reliability [13].

7 Conclusion

Carlos Kenig's work exemplifies originality, depth, lasting influence, and meaningful contribution to human knowledge. Through research that reshaped the modern theory of PDEs, he established new ways of understanding complexity in mathematical science. His contributions addressed nonlinearity and irregularity and provided unified analytical frameworks that are now indispensable across mathematics and the applied sciences.

Equally significant is the enduring nature of his influence: his ideas continue to shape research directions, guide new methodologies, and inform interdisciplinary collaboration. At a time when science increasingly depends on complex models and large-scale computation, Kenig's emphasis on rigorous structure and minimal assumptions offers a stabilising foundation for future innovation. For these reasons, his scientific legacy aligns fully with the mission of the King Faisal Prize.

References

[1] L. C. Evans, *Partial Differential Equations*, 2nd ed., American Mathematical Society, 2010.

[2] A. Majda and A. Bertozzi, *Vorticity and Incompressible Flow*, Cambridge University Press, 2002.

[3] C. E. Kenig, *Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems*, CBMS Regional Conference Series in Mathematics, AMS, 1994.

[4] C. E. Kenig, "Some recent applications of harmonic analysis to partial differential equations," in *Proceedings of the International Congress of Mathematicians*, 2006.

[5] L. A. Caffarelli, "The obstacle problem revisited," *Journal of Fourier Analysis and Applications*, 4 (1998), 383–402.

[6] D. Jerison and C. E. Kenig, "Boundary value problems on Lipschitz domains," in *Studies in Partial Differential Equations*, MAA Studies in Mathematics, 23 (1982), 1–68.

[7] H. W. Alt, L. A. Caffarelli, and A. Friedman, "Variational problems with two phases and their free boundaries," *Transactions of the American Mathematical Society*, 282 (1984), 431–461.

[8] C. E. Kenig and T. Toro, "Free boundary regularity for harmonic measures and Poisson kernels," *Annals of Mathematics*, 150 (1999), 369–454.

[9] E. M. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton University Press, 1993.

[10] P. Constantin and C. Foias, *Navier–Stokes Equations*, University of Chicago Press, 1988.

[11] J. Rauch, *Partial Differential Equations*, Springer, 1991.

[12] V. Isakov, *Inverse Problems for Partial Differential Equations*, 2nd ed., Springer, 2006.

[13] L. Ambrosio, N. Fusco, and D. Pallara, *Functions of Bounded Variation and Free Discontinuity Problems*, Oxford University Press, 2000.

[14] M. Raissi, P. Perdikaris, and G. E. Karniadakis, "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations," *Journal of Computational Physics*, 378 (2019), 686–707.

[15] G. E. Karniadakis, I. G. Kevrekidis, L. Lu, P. Perdikaris, S. Wang, and L. Yang, "Physics-informed machine learning," *Nature Reviews Physics*, 3 (2021), 422–440.

[16] J.-L. Lions, R. Temam, and S. Wang, "Models of the coupled atmosphere and ocean," *Computational Mechanics*, 9 (1992), 1–14.